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An extended Riccati sub-equation method to establish new exact solutions for fraction nonlinear differentional difference lattice equations

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Abstract— The main objective of this paper is to apply the Riccati sub-equation method to establish new exact solutions for the nonlinear component fractional Hybrid lattice Equation. As a result, new traveling wave solutions including hyperbolic function solutions, trigonometric function solutions and rational function solutions are obtained. Our solutions can be viewed as a generalization to the results which found in some recent published papers.

Index Terms— Nonlinear differential-difference equation, extended Riccati subequation fractional sub-equation method.

1 Introduction

Nonlinear differential difference equations (NDDEs) play a crucial role in many branches of applied physical sciences such as condensed matter physics, biophysics, atomic chains, molecular crystals, and discretization in solid-state and quantum physics. They also play an important role in numerical simulation of soliton dynamics in high-energy physics because of their rich structures. Therefore, researchers have shown a wide interest in studying NDDEs since the original work of Fermi et al. [1] in the 1950s. Contrary to difference equations that are being fully discretized, NDDEs are semidiscretized, with some (or all) of their space variables being discretized, while time is usually kept continuous. As far as we could verify, little work has been done to search for exact solutions of NDDEs. Hence, it would make sense to do more research on solving NDDEs. [2-3]. Many analytic approximate approaches for solving nonlinear differential equations have been proposed and the most outstanding one is the homotopy analysis method (HAM) and fractional sub-equation method . In recent years, many authors have paid attention to studying the solutions of nonlinear partial differential equations and nonlinear differential difference equations by various methods [4-20].

The main objective of the present paper is to apply fractional sub-equation method to establish new exact solutions for the nonlinear component fractional lattice Equation

$0\le\eta<1$, $u_n=u_n(t)$ and α,β,γ are constants.

As a result, new traveling wave solutions including hyperbolic function solutions, trigonometric function solutions and rational function solutions are obtained. Our solutions can be viewed as a generalization to the results which found in some recent published papers. In the next of this section, we give some definitions and properties of the modified Riemann- Liouville derivative [21] which are used in this paper. Assume that $f:R\to R, x\mapsto f(x)$ denote a continous (but not necessarily differentiable) function, and let h denote a constant discretization span, Jumarie's defined the fractional derivative in the limit form

$$f^{\alpha}(x)=\lim_{h\downarrow 0}\frac{\Delta^{\alpha}\left[f(x)-f(0)\right]}{h^{\alpha}},0<\alpha<1,$$

Where

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$$\Delta^{\alpha} f(x)=\sum_{k=0}^{\infty}(-1)^{k}\frac{\Gamma(1+\alpha)}{\Gamma(1+k)\Gamma(\alpha-k+1)} f\left[x+(\alpha-k)h\right].$$

This definition is close to the standard definition of the derivative (calculus for beginners) and as a direct result, the $\alpha-$ th derivative of a constant, $0<\alpha<1$, is zero. An alternative, which is the strictly equivalent to Eq. (1) is the following expression as

$$f^{\alpha}(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_{0}^{x}(x-\xi)^{-\alpha}\left[f(\xi)-f(0)\right]d\xi, 0<\alpha<1$$

and

$$f^{\alpha}(x)=\left(f^{(n)}(x)\right)^{(\alpha-n)}, n\le\alpha\le n+1, n\ge 1.$$

Some properties of the fractional modified Riemann-Liouville derivative were summarized in four useful formulas of them are

$$D_x^{\alpha} x^{\gamma}=\frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)}x^{\gamma-\alpha}, \gamma>0,$$

$$D_x^{\alpha}\left(u(x)v(x)\right)=v(x)D_x^{\alpha}u(x)+u(x)D_x^{\alpha}v(x),$$

$$D_x^{\alpha}\left[f(u(x))\right]=f_u^{'}(u)D_x^{\alpha}u(x),$$

$$D_x^{\alpha}\left[f(u(x))\right]=D_u^{\alpha}f(u)\left(u_x^{'}\right)^{\alpha}.$$

The rest of this paper is organized as follows. In Sec. 2, we give the description of the proposed method. Then in Sec. 3 we apply the method to find exact solutions for the non linear component fractional lattice Equation

$$D_t^{\eta} u_n=\left(\alpha+\beta u_n+\gamma u_n^2\right)\left(u_{n-1}-u_{n+1}\right),$$ where $0\le\eta<1$, $u_n=u_n(t)$ and α,β,γ are constants . Some conclusions are presented at the end of the paper.

2 Description of the Extended Riccati Sub-Equation Method

The main steps of the extended Riccati subequation method for solving nonlinear lattice equations are summarized as follows:

Step 1. Consider a system of M polynomial nonlinear lattice equations in the form

$$P\begin{pmatrix} u_{n+p_1}(x), u_{n+p_2}(x),\ldots,u_{n+p_k}(x),u'_{n+p_1}(x), \\ u'_{n+p_2}(x),\ldots,u'_{n+p_k}(x),\ldots,u^{(r)}_{n+p_k}(x)\end{pmatrix}=0 \quad (2)$$

where the dependent variable u has M components u_i, the continuous variable x has N components x_j, the discrete variable n has Q components n_i, the k shift vectors $p_s\in\mathbb{Z}^Q$ has Q components p_{s_j}, and $u^{(r)}(x)$ denotes the collection of mixed derivative terms of order r.

Step 2. Using a wave transformation

$$u_{n+p_s}(x)=U_{n+p_s}(\xi_n), \xi_n=\sum_{i=1}^{Q}d_i n_i+\sum_{j=1}^{N}c_j x_j+\zeta$$

where d_i, c_j, ζ are all constants, we can rewrite Eq. (2) as the following nonlinear form:

$$P\begin{pmatrix} U_{n+p_1}(\xi_n),U_{n+p_2}(\xi_n),\ldots,U_{n+p_k}(\xi_n),U'_{n+p_1}(\xi_n), \\ U'_{n+p_2}(\xi_n),\ldots,U'_{n+p_k}(\xi_n),\ldots,U^{(r)}_{n+p_1}(\xi_n),U^{(r)}_{n+p_2}(\xi_n),\ldots,U^{(r)}_{n+p_k}(\xi_n)\end{pmatrix}=0 \quad (3)$$

Step 3. Suppose the solutions of Eq. (3) can be denoted by

$$u_n(x)=U(\xi_n)=\sum_{i=0}^{l}a_i\,\phi^i(\xi_n) \quad (4)$$

where a_i are constants to be determined later, l is a positive integer that can be determined by balancing the highest order linear term with the nonlinear terms in Eq. (3), $\phi(\xi_n)$ satisfies the known Riccati equation:

$$\frac{d\phi_n(\xi_n)}{d\xi_n}=\sigma+\phi_n^2(\xi_n) \quad (5)$$

Step 4. We present some special solutions $\phi_1,\phi_2,\ldots,\phi_6$ for Eq. (5):

When $\sigma<0$,

$$\phi_1(\xi_n)=-\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\xi_n+c_0\right),$$

$$\phi_2(\xi_n)=-\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}\xi_n+c_0\right),$$

$$\phi_{1,2}\left(\xi_{n+p_s}\right)=\frac{\phi_{1,2}\left(\xi_n\right)-\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\sum_{i=1}^{Q}d_i p_{s_i}\right)}{1-\frac{\phi_{1,2}\left(\xi_n\right)}{\sqrt{-\sigma}}\tanh\left(\sqrt{-\sigma}\sum_{i=1}^{Q}d_i p_{s_i}\right)} \quad (6)$$

where c_0 is an arbitrary constant.

When $\sigma>0$,

$$\phi_3\left(\xi_n\right)=\sqrt{\sigma}\tan\left(\sqrt{\sigma}\xi_n+c_0\right),$$

$$\phi_2\left(\xi_n\right)=-\sqrt{\sigma}\cot\left(\sqrt{\sigma}\xi_n+c_0\right),$$

$$\phi_{3,4}\left(\xi_{n+p_s}\right)=\frac{\phi_{3,4}\left(\xi_n\right)+\sqrt{\sigma}\tan\left(\sqrt{\sigma}\sum_{i=1}^{Q}d_i p_{s_i}\right)}{1-\frac{\phi_{3,4}\left(\xi_n\right)}{\sqrt{\sigma}}\tan\left(\sqrt{\sigma}\sum_{i=1}^{Q}d_i p_{s_i}\right)} \quad (7)$$

and

$$\phi_5\left(\xi_n\right)=\sqrt{\sigma}\left[\tan\left(2\sqrt{\sigma}\xi_n+c_0\right)+\left|\sec\left(2\sqrt{\sigma}\xi_n+c_0\right)\right|\right],$$

$$\phi_5\left(\xi_{n+p_s}\right)=\frac{\phi_5^{(1)}\left(\xi_n\right)+\sqrt{\sigma}\left(\tan\left(2\sqrt{\sigma}\sum_{i=1}^{Q}d_i p_{s_i}\right)\right)}{1-\frac{\phi_5^{(1)}\left(\xi_n\right)}{\sqrt{\sigma}}\left(\tan\left(2\sqrt{\sigma}\sum_{i=1}^{Q}d_i p_{s_i}\right)\right)}+\frac{\phi_5^{(2)}\left(\xi_n\right)+\sqrt{\sigma}\left(\sec\left(2\sqrt{\sigma}\sum_{i=1}^{Q}d_i p_{s_i}\right)\right)}{1-\frac{\phi_5^{(1)}\left(\xi_n\right)}{\sqrt{\sigma}}\left(\tan\left(2\sqrt{\sigma}\sum_{i=1}^{Q}d_i p_{s_i}\right)\right)} \quad (8)$$

Where

$$\phi_5^{(1)}\left(\xi_n\right)=\sqrt{\sigma}\left(\tan\left(2\sqrt{\sigma}\xi_n+c_0\right)\right),$$

$$\phi_5^{(2)}\left(\xi_n\right)=\sqrt{\sigma}\left(\sec\left(2\sqrt{\sigma}\xi_n+c_0\right)\right),$$

and c_0 is an arbitrary constant.

when $\sigma=0$,

$$\phi_6\left(\xi_n\right)=-\frac{1}{\xi_n+c_0},$$

$$\phi_6\left(\xi_{n+p_s}\right)=\frac{\phi_6\left(\xi_n\right)}{1-\phi_6\left(\xi_n\right)\sum_{i=1}^{Q}d_i p_{s_i}} \quad (9)$$

and c_0 is an arbitrary constant.

Step 5. Substituting (4) into Eq. (3), by use of Eqs. (5)-(9), the left hand side of Eq. (3) can be converted into a polynomial in $\phi\left(\xi_n\right)$. Equating each coefficient of $\phi^i\left(\xi_n\right)$ to zero, yields a set of algebraic equations. Solving these equations, we can obtain the values of a_i, d_i, c_j .

Step 6. Substituting the values of a_i into (4), and combining with the various solutions of Eq.(5), we can obtain a variety of exact solutions for Eq. (2).

2 Application of the Extended Riccati Sub-Equation Method

In this section, we will apply the described method to the fractional Hybrid lattice equation:

$$D_t^{\eta}u_n=\left(\alpha+\beta u_n+\gamma u_n^2\right)\left(u_{n-1}-u_{n+1}\right),0\le\eta<1,$$

Using a fractional complex transformation $T=\frac{x^{\eta}}{\Gamma(1+\eta)}$ and letting $u_{n+p_s}(x)=U_{n+p_s}\left(\xi_n\right), p_s=0,\pm1,$ we have $D_x^{\eta}T=1$. Letting $U_n(T)=\bar{U}_n\left(\xi_n\right), \xi_n=d_1 n+c_1 T+\zeta$, where d_1, c_1, ζ are all constants, Eq. (2) can be rewritten in the following form:

$$c_1\bar{U}_n'\left(\xi_n\right)=\left(\alpha+\beta\bar{U}_n\left(\xi_n\right)+\gamma\bar{U}_n^2\left(\xi_n\right)\right)x\left(\bar{U}_{n-1}\left(\xi_n\right)-\bar{U}_{n+1}\left(\xi_n\right)\right) \quad (10)$$

Suppose the solutions of Eq. (10) can be denoted by

$$\bar{U}_n\left(\xi_n\right)=\sum_{i=0}^{l}a_i\phi^i\left(\xi_n\right) \quad (11)$$

where $\phi\left(\xi_n\right)$ satisfies Eq. (5). Balancing the order of $\bar{U}_n'\left(\xi_n\right)$ and $\bar{U}_n^2\left(\xi_n\right)$ in Eq. (10) we obtain $l+1=2l$, and then $l=1$. So we have

$$\bar{U}_n\left(\xi_n\right)=\sum_{i=0}^{1}a_i\phi^i\left(\xi_n\right)=a_0+a_1\phi\left(\xi_n\right). \quad (12)$$

We will proceed to solve Eq. (2) in several cases.

Case 1. If $\sigma<0$, and assume (5) and (6) hold, then substituting (12), (5) and (6) into Eq. (10), collecting the coefficients of $\phi_{1,2}^i\left(\xi_n\right)$ and equating them to zero, we obtain a series of algebra equations. Solving these equations, yields

$$a_1 = \pm\sqrt{\frac{4\alpha\gamma-\beta^2}{\sigma}}\frac{\tanh\left(\sqrt{-\sigma}d_1\right)}{2\gamma},$$

$$a_0 = -\frac{\beta}{2\gamma}, d_1 = d_1, c_1 = \frac{\beta^2-4\alpha\gamma}{2\sqrt{-\sigma}\gamma}\tanh\left(\sqrt{-\sigma}d_1\right), \beta^2-4\alpha\gamma > 0$$

So we obtain the following solitary wave solutions:

$$\bar{U}_n\left(\xi_n\right) = -\frac{\beta}{2\gamma} \pm \sqrt{\beta^2-4\alpha\gamma}\frac{\tanh\left(\sqrt{-\sigma}d_1\right)}{2\gamma}\times$$
$$\tanh\left[\sqrt{-\sigma}\left(d_1 n + \frac{\beta^2-4\alpha\gamma}{2\sqrt{-\sigma}\gamma}\tanh\left(\sqrt{-\sigma}d_1\right)t+\zeta\right)+c_0\right],$$

$$\bar{U}_n\left(\xi_n\right) = -\frac{\beta}{2\gamma} \pm \sqrt{\beta^2-4\alpha\gamma}\frac{\tanh\left(\sqrt{-\sigma}d_1\right)}{2\gamma}\times$$
$$\coth\left[\sqrt{-\sigma}\left(d_1 n + \frac{\beta^2-4\alpha\gamma}{2\sqrt{-\sigma}\gamma}\tanh\left(\sqrt{-\sigma}d_1\right)t+\zeta\right)+c_0\right]$$

$$\xi_n = d_1 n + \frac{\beta^2-4\alpha\gamma}{2\sqrt{-\sigma}\gamma}\tanh\left(\sqrt{-\sigma}d_1\right)\frac{x^{\eta}}{\Gamma(1+x)}+\zeta$$

Case 2. If $\sigma > 0$, and assume (5) and (7) hold, then substituting (12), (5) and (7) into Eq. (10), collecting the coefficients of $\phi_{1,2}^i\left(\xi_n\right)$ and equating them to zero, we obtain a series of algebra equations. Solving these equations, yields

$$a_1 = \pm\sqrt{\frac{\beta^2-4\alpha\gamma}{\sigma}}\frac{\tan\left(\sqrt{-\sigma}d_1\right)}{2\gamma},$$

$$a_0 = -\frac{\beta}{2\gamma}, d_1 = d_1, c_1 = \frac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}\tan\left(\sqrt{\sigma}d_1\right), \beta^2-4\alpha\gamma > 0$$

Then we have the following trigonometric function solutions:

$$\bar{U}_n\left(\xi_n\right) = -\frac{\beta}{2\gamma} \pm \sqrt{\beta^2-4\alpha\gamma}\frac{\tan\left(\sqrt{\sigma}d_1\right)}{2\gamma}\times$$
$$\tan\left[\sqrt{\sigma}\left(d_1 n + \frac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}\tanh\left(\sqrt{\sigma}d_1\right)t+\zeta\right)+c_0\right],$$

$$\bar{U}_n\left(\xi_n\right) = -\frac{\beta}{2\gamma} \pm \sqrt{\beta^2-4\alpha\gamma}\frac{\tan\left(\sqrt{\sigma}d_1\right)}{2\gamma}\times$$
$$\cot\left[\sqrt{\sigma}\left(d_1 n + \frac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}\tanh\left(\sqrt{\sigma}d_1\right)t+\zeta\right)+c_0\right]$$

$$\xi_n = d_1 n + \frac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}\tan\left(\sqrt{\sigma}d_1\right)\frac{x^{\eta}}{\Gamma(1+x)}+\zeta$$

Case 3. If $\sigma > 0$, and assume (5) and (8) hold, then substituting (12), (5) and (8) into Eq. (10), using $\left[\phi_5^{(2)}\left(\xi_n\right)\right]^2 = \sigma + \left[\phi_5^{(1)}\left(\xi_n\right)\right]^2$ collecting the coefficients of $\left[\phi_5^{(2)}\left(\xi_n\right)\right]^i\left[\phi_5^{(1)}\left(\xi_n\right)\right]^j\phi_{1,2}^i\left(\xi_n\right)$ and equating them to zero, we obtain a series of algebra equations. Solving these equations, yields

$$a_1 = a_1, a_0 = -\frac{\beta}{2\gamma}, d_1 = \pm\frac{\pi}{4\sqrt{\sigma}},$$

$$c_1 = \pm 2\sqrt{\sigma}\gamma a_1^2, \beta^2-4\alpha\gamma > 0$$

or

$$a_1 = \sqrt{\frac{\beta^2-4\alpha\gamma}{\sigma}}\frac{1}{2\gamma}, a_0 = -\frac{\beta}{2\gamma}, d_1 = \pm\frac{\pi}{4\sqrt{\sigma}},$$

$$c_1 = \pm\frac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}, \beta^2-4\alpha\gamma > 0$$

or

$$a_1 = -\sqrt{\frac{\beta^2-4\alpha\gamma}{\sigma}}\frac{1}{2\gamma}, a_0 = -\frac{\beta}{2\gamma}, d_1 = \pm\frac{\pi}{4\sqrt{\sigma}},$$

$$c_1 = \pm\frac{\beta^2-4\alpha\gamma}{2\sqrt{\sigma}\gamma}, \beta^2-4\alpha\gamma > 0$$

or

$$a_1 = a_1, a_0 = -\frac{\beta}{2\gamma}, d_1 = \frac{1}{2\sqrt{\sigma}}\arccos\left(\frac{-\beta^2+4\alpha\gamma+4\sigma\gamma^2a_1^2}{-\beta^2+4\alpha\gamma-4\sigma\gamma^2a_1^2}\right)\pm\frac{\pi}{4},$$

$$c_1 = \pm a_1\sqrt{\beta^2-4\alpha\gamma}, \beta^2-4\alpha\gamma > 0$$

So we obtain the following four groups of trigonometric function solutions:

$$\bar{U}_n\left(\xi_n\right) = a_1\left\{\begin{array}{l}\tan\left[2\sqrt{\sigma}\left(\pm\frac{\pi}{4\sqrt{\sigma}}\pm 2\sqrt{\sigma}\gamma a_1^2+\zeta\right)+c_0\right]+ \\ \left|\sec\left[2\sqrt{\sigma}\left(\pm\frac{\pi}{4\sqrt{\sigma}}\pm 2\sqrt{\sigma}\gamma a_1^2+\zeta\right)+c_0\right]\right|\end{array}\right\}-\frac{\beta}{2\gamma}$$

$$\bar{U}_n(\xi_n)=a_1\left\{\begin{array}{l}\tan\left[2\sqrt{\sigma}\left(\begin{array}{l}\frac{1}{2\sqrt{\sigma}}\arccos\left(\frac{-\beta^2+4\alpha\gamma+4\sigma\gamma^2a_1^2}{-\beta^2+4\alpha\gamma-4\sigma\gamma^2a_1^2}\right)n\\ \pm a_1\sqrt{\beta^2-4\alpha\gamma}t+\zeta\end{array}\right)+c_0\right]\\ +\\ \sec\left[2\sqrt{\sigma}\left(\begin{array}{l}\frac{1}{2\sqrt{\sigma}}\arccos\left(\frac{-\beta^2+4\alpha\gamma+4\sigma\gamma^2a_1^2}{-\beta^2+4\alpha\gamma-4\sigma\gamma^2a_1^2}\right)n\\ \pm a_1\sqrt{\beta^2-4\alpha\gamma}+\zeta\end{array}\right)+c_0\right]\end{array}\right\}-\frac{\beta}{2\gamma}$$

$$\bar{U}_n(\xi_n)=\frac{\sqrt{\beta^2-4\alpha\gamma}}{2\gamma}\left\{\begin{array}{l}\tan\left[2\sqrt{\sigma}\left(\pm\frac{\pi}{4\sqrt{\sigma}}n\pm\frac{\sqrt{\beta^2-4\alpha\gamma}}{2\gamma\sqrt{\sigma}}t+\zeta\right)+c_0\right]+\\ \sec\left[2\sqrt{\sigma}\left(\pm\frac{\pi}{4\sqrt{\sigma}}n\pm\frac{\sqrt{\beta^2-4\alpha\gamma}}{2\gamma\sqrt{\sigma}}t+\zeta\right)+c_0\right]\end{array}\right\}-\frac{\beta}{2\gamma}$$

$$\bar{U}_n(\xi_n)=-\frac{\sqrt{\beta^2-4\alpha\gamma}}{2\gamma}\left\{\begin{array}{l}\tan\left[2\sqrt{\sigma}\left(\pm\frac{\pi}{4\sqrt{\sigma}}n\pm\frac{\sqrt{\beta^2-4\alpha\gamma}}{2\gamma\sqrt{\sigma}}t+\zeta\right)+c_0\right]+\\ \sec\left[2\sqrt{\sigma}\left(\pm\frac{\pi}{4\sqrt{\sigma}}n\pm\frac{\sqrt{\beta^2-4\alpha\gamma}}{2\gamma\sqrt{\sigma}}t+\zeta\right)+c_0\right]\end{array}\right\}-\frac{\beta}{2\gamma}$$

where a_1, c_0 are arbitrary constants, and $a_1 \neq 0$.

Case 4. If $\sigma = 0$, and assume (5) and (9) hold, then substituting (31), (5) and (9) into Eq. (10), collecting the coefficients of $\phi^i(\xi_n)$ and equating them to zero, we obtain the following solutions:

Then we have the following trigonometric function solutions:

$$a_1=\pm\frac{\sqrt{\beta^2-4\alpha\gamma}d_1}{2\gamma}, a_0=-\frac{\beta}{2\gamma}, d_1=d_1,$$
$$c_1=\pm\frac{(\beta^2-4\alpha\gamma)d_1}{2\gamma}, \beta^2-4\alpha\gamma>0.$$

Then we obtain the following rational solution:

$$\bar{U}_n(\xi_n)=a_1=\pm\frac{\sqrt{\beta^2-4\alpha\gamma}d_1}{2\gamma}\times\left(\frac{1}{d_1n+\frac{(\beta^2-4\alpha\gamma)d_1}{2\gamma}t+\zeta+c_0}\right)-\frac{\beta}{2\gamma}$$

where d_1, c_0 are arbitrary constants.

4. Conclusions

In this paper, the fractional sub-equation method has been successfully applied for establish new exact solutions for the nonlinear component fractional lattice equations. The result shows that the fractional sub-equation is a powerful and efficient technique in finding exact solutions for nonlinear differential equations. In conclusion, the fractional sub-equation method may be considered a nice refinement in finding the exact solutions for the nonlinear component fractional lattice equations existing and may find wide applications.

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